

# FERMION NUMBER VIOLATION IN HEAVY FERMION DECAY

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## ABSTRACT

The presence of a heavy fermion doublet of fourth generation in the standard model would lead to a anomalous decay with fermion number non-conservation. The anomalous decay path in the background of the electroweak instanton is demonstrated by numerical calculation. If the mass of fermion exceeds the critical value  $m_f^{\text{cr}}=9.2$  TeV (for  $M_H = M_W$ ), it is shown that there exists the rapid anomalous decay of the heavy fermion in semi-classical approximation. The dependence of the critical fermion mass on the Higgs mass is also presented.

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## I. INTRODUCTION

Recent experimental observation for top quark with large mass around 176 GeV has been reported by CDF Collaboration[1]. The top quark mass is much bigger than the masses of other quarks. There is a large mass hierarchy among three generation fermions. The ratio of  $m_u:m_c:m_t$  is approximately as 1:260:35200. On the other hand, present experimental accuracy is not sufficient to exclude the existence of one extra generation of leptons and quarks[2]. The new fermions are supposed to possess the same colour and electroweak quantum numbers as the ordinary ones and to mix very tiny with the ordinary three generation. If the fourth generation quarks exist, it would be very heavy according to the large mass ratio for up, charm and top. The mass of fourth generation quarks would be over 10 TeV because the mass ratio between two generations is always over 100 times. So, it is very interesting to investigate the physics of standard model with very heavy fermion.

When the heavy fourth generation quarks represents in the standard model, one of the most important effects is that the non-perturbative electroweak fermion number non-conservation may be naturally unsuppressed[3]. In this paper we consider the anomalous decay of the heavy fourth generation quarks in standard electroweak model. We are known that the baryon number and lepton number do not absolutely conserve in the standard model due to the quantum anomaly[4]. The process for baryon number and lepton number nonconservation is spontaneous fermion number violation due to instanton induced transitions between topologically distinct vacua[5]. Under normal conditions, the amplitudes with nonconserved baryon number that are due to this mechanism are suppressed by the factors  $\exp(-8\pi^2/g_W^2) \sim 10^{-170}$ , where  $g_W^2 = e^2/\sin^2 \theta_W$  is the coupling constant of the electroweak gauge group  $SU(2)_L$ . However, it has recently been realized that there can be a great amplification of anomalous fermion number nonconservation[6]. Generally, this might occur when the energy stored in the system is big enough. In principle, the energy can be of different forms. The simplest condition is provided by the system at high-temperatures[7], or collisions of particles at high-energies[8], and in decays of heavy particles[9]. The characteristic energy scale, at which the anomalous process become rapid, is set by the sphaleron mass, which determines the height of the energy barrier between the topologically distinct vacua, and is of order 10 TeV.

The mechanism of fermion number nonconservation can be directly shown in the

fermion level crossing picture[10,11]. When the bosonic field configuration evolves from the trivial vacuum configuration to the topologically distinct one with unit topological number, the fermion energy level emerges from the positive continuum, crosses zero and disappears in the negative continuum. At small fermion masses, the state corresponding to the fermion is separated from the topologically non-trivial vacuum state by a finite energy barrier. The fermion can decay into the vacuum state due to semi-classical tunneling through this barrier, but the probability of such a process is exponentially small. If the mass of the fermion is comparable with the height of the energy barrier, the fermion may become classically unstable with respect to the anomalous decay. In this case the barrier between the fermion and the topologically non-trivial vacuum disappears, the decay, instead of tunneling, proceeds classically and the exponential suppression of the amplitude is absent.

Anomalous fermion number non-conservation in decays of the system of elementary fermions has been investigated by Rubakov[3] at first, It was shown that the mass of fermion exceeds some critical values, the barrier disappears and the system freely rolls down, decaying into the vacuum state with zero fermion number. Recently, Petriashvili [9] has calculated the critical values of fermion mass by using the variational ansatz. In this paper, we will demonstrate the whole path of anomalous decay of heavy fermion in an electroweak instanton by numerical calculation and give the critical values of fermion mass for unsuppressed fermion number non-conservation appearing. For simplicity, we assume that the charged and neutral components of the fourth generation doublet have equal masses and neglect the weak hypercharge interactions  $U(1)$ . Throughout this paper we work in the classical approximation, i.e. we neglect the radiative corrections due to the boson loops and the contribution of the Dirac sea to the energy of the system. For strong Yukawa coupling, this approximation is far from being convincing, nevertheless, we do hope that our results are qualitatively correct.

On the other hand, the heavy fermion in the standard model cannot be included without violating vacuum stability[12]. However, the problem may be overcome by introducing new physics in the TeV region, for example, the arguments of ref. 12 are not valid if there exist heavy scalars with roughly the same masses, as can be the case in supersymmetric theories. Therefore, we think that the decays of heavy fourth generation quarks can be of phenomenological interest.

We work in the  $A_0(x) = 0$  gauge and focus upon the Euclidean time parameter  $x_0$ . As  $x_0$  changes from  $-\infty$  to  $+\infty$ , the one-instanton field evolves from one pure gauge configuration to another topologically distinct pure gauge. The three-dimensional Dirac Hamiltonian in the presence of such a field depends parametrically on  $x_0$  through

$A_i(x_0, \mathbf{x})$ . The spectrum of eigenenergy of fermion as a function of  $x_0$ , gives information about the behavior of the quantized Dirac field in the adiabatic approximation. Then, we can calculate the total energy of the system which depend on  $x_0$ . If the total energy monotonically decreases (energy barrier is absent), one can expect that rapid anomalous decay of fermion might occur classically.

The remainder of this paper is organized as follows. Section II introduce the Weinberg-Salam model Lagrangian with two approximations employed and the formulas of fermion number violation in electroweak theory. The electroweak instanton solution and its expression in the Minkowski space are given in Sec. III. In this section, the Chern-Simons number and the static energy of the instanton are presented as a function of Euclidean time. In Sec. IV we derive the radial equations for fermions. We present the anomalous decay path of the heavy fermion in the background of instantons by numerical calculation in Sec. V. Finally, in Sec. VI, a brief discussion is given.

## II. FERMION NUMBER VIOLATION

Let us consider the bosonic sector of the Weinberg-Salam model in the limit of vanishing mixing angle. In this limit the  $U(1)$  field decouples and can consistently be set to zero:

$$\mathcal{L}_b = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu,a} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda(\Phi^\dagger \Phi - \frac{v^2}{2})^2 \quad (2.1)$$

with the  $SU(2)_L$  field strength tensor,

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c, \quad (2.2)$$

and the covariant derivative for the Higgs field,

$$D_\mu \Phi = (\partial_\mu - \frac{i}{2}g\tau^a A_\mu^a)\Phi, \quad (2.3)$$

where  $g$  is the gauge coupling constant and in electroweak theory we employ the value  $g = 0.67$ .  $A_\mu^a(x)$  ( $a = 1, 2, 3$ ,) are real vector fields and can be described as a matrix field  $A_\mu(x) = \frac{1}{2}g\tau^a A_\mu^a(x)$ ,  $\tau^a$  being the isospin Pauli matrices.

The  $SU(2)_L$  gauge symmetry is spontaneously broken due to the non-vanishing vacuum expectation value  $v$  of the Higgs field,

$$\langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.4)$$

leading to the boson masses

$$M_W = M_Z = \frac{1}{2}gv, \quad M_H = v\sqrt{2\lambda}, \quad (2.5)$$

where we take  $v=246\text{GeV}$ .

The model has a non-trivial vacuum structure. The classical vacuum configuration, being defined by the minima of the bosonic part of static energy

$$E_b = \int d^3x \left\{ -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu,a} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \lambda(\Phi^\dagger \Phi - \frac{v^2}{2})^2 \right\} \quad (2.6)$$

are represented by the pure gauge configurations

$$A_\mu(x) = U(x)\partial_\mu U^{-1}(x), \quad \Phi(x) = U(x)\phi_0, \quad (2.7)$$

where  $U(x)$  is a  $2 \times 2$  unitary matrix of  $\text{SU}(2)$ ;  $\phi_0$  is a constant isospinor and takes the value of the Higgs field for  $A_\mu = 0$ . The classical vacua have a discrete set labeled by the integer  $q$ , and one vacuum differs from another topologically by nontrivial gauge transformations which carry an topological number

$$q = \frac{g^2}{16\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a. \quad (2.8)$$

The topological current is

$$K^\mu = \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr}(A_\nu \partial_\rho A_\sigma - i\frac{2}{3}g A_\nu A_\rho A_\sigma). \quad (2.9)$$

All gauge field configurations can be classified by the Chern-Simons number given by

$$N_{CS} = \int d^3x K^0. \quad (2.10)$$

The Chern-Simons number  $N_{CS}$  may be regarded as a coordinate in gauge-orbit space which measures the position of topologically inequivalent vacua. For the vacua the Chern-Simons number is identical to the integer, for nonvacuum it may take on arbitrary real values. Neighbouring vacua with different topological numbers are separated by a finite energy barrier. The height of the barrier is determined by the sphaleron solution of the bosonic field equations[13]. This saddle-point solution has Chern-Simons number  $N_{CS} = 1/2$ , and its energy is equal to the height of the barrier.

The most straightforward realization of a fermionic extension of the standard model is the introduction of a fourth generation of fermions. The new fermions are supposed

to possess the same colour and electroweak quantum numbers as the ordinary ones. For vanishing mixing angle, considering the heavy fermion doublet degenerate in mass, the heavy fermion Lagrangian in the chiral representation reads,

$$\begin{aligned}\mathcal{L}_f = & \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{u}_R i\gamma^\mu \partial_\mu u_R + \bar{d}_R i\gamma^\mu \partial_\mu d_R \\ & - f_q \bar{q}_L (\tilde{\Phi} u_R + \Phi d_R) - f_q (\bar{d}_R \Phi^\dagger + \bar{u}_R \tilde{\Phi}^\dagger) q_L,\end{aligned}\tag{2.11}$$

where  $q_L$  denotes the lefthanded doublet  $(u_L, d_L)$ ,  $u_R$  and  $d_R$  are the righthanded singlets, with covariant derivative,

$$D_\mu q_L = (\partial_\mu - \frac{i}{2} g \tau^a A_\mu^a) q_L,\tag{2.12}$$

and with  $\tilde{\Phi} = i\tau_2 \Phi^*$ . The heavy fermion mass is given by

$$m_u = m_d = m_f = \frac{1}{\sqrt{2}} f_q v.\tag{2.13}$$

The gauge-invariant current of the doublet  $J^\mu = \bar{q}_L \gamma^\mu q_L$  is conserved at the classical level, but is anomalous at the quantum level:

$$\partial_\mu J^\mu = \frac{g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a.\tag{2.14}$$

The integration of right-hand side of the equation (2.14) is expression of the topological charge of a gauge field configuration as (2.8). The equation (2.14) indicates that the number of fermions may not be conserved, the changes of baryon number  $B$  and lepton number  $L$  are given as

$$\Delta B = \Delta L = n_f q,\tag{2.15}$$

where  $n_f$  is the number of generations.

### III. EXPRESSION OF INSTANTON IN MINKOWSKI SPACE

The fermion number non-conservation in the electroweak theory is associated with instanton which takes Euclidean field configuration[14]. For  $SU(2)$  gauge field, an explicit solutions with topological charge  $q = 1$  in the regular gauge is given by

$$\begin{aligned}A_0(x) &= -\frac{i\boldsymbol{\tau} \cdot \mathbf{x}}{x^2 + \rho^2}, \\ \mathbf{A}(x) &= -\frac{i(\tau x_0 + \boldsymbol{\tau} \times \mathbf{x})}{x^2 + \rho^2},\end{aligned}\tag{3.1}$$

where  $x^2 = x_0^2 + \mathbf{x}^2$  and  $\rho$  is some arbitrary scale parameter, often referred to as the instanton size. The instanton can be viewed as a solution of the Euclidean gauge field equations in which a vacuum at  $x_0 = -\infty$  evolves by propagation in imaginary time to a different vacuum at  $x_0 = +\infty$ .

In the Weinberg-Salam model, due to the presence of the Higgs field, strictly speaking, for  $v \neq 0$ , there does not exist a finite action solution of the classical Euclidean equations of motion. However, as long as the size of instanton  $\rho$  is not too large compared to the inverse size of the order parameter  $v$  in the Weinberg-Salam model, as  $\rho v \ll 1$ , one can find a good approximate solution in the Weinberg-Salam model[15]. This approximate solution still has the gauge field configurations given by the  $SU(2)$  instanton of (3.1), while the Higgs doublet field takes the form,

$$\Phi = \frac{x_0 - i\tau \cdot \mathbf{x}}{\sqrt{x^2 + \rho^2}} \frac{v}{\sqrt{2}} \Phi_0 \quad (3.2)$$

where  $\Phi_0$  is a constant  $SU(2)$  spinor and we can take  $\Phi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  and  $\tilde{\Phi}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

In the Minkowski space version for instanton, it describes the tunneling process in potential barrier which connects one vacuum to another vacuum. In the gauge  $A_0 = 0$ , the instanton determines the path in the configuration space connecting the vacua with different topological number,  $x_0$  being the parameter along the path. If the starts from the trivial gauge vacuum ( $N_{CS}=0$ ) with filled Dirac sea plus one real fermion, and evolves to the non-trivial gauge vacuum with  $N_{CS} = 1$ , then the final state has filled Dirac sea and no real fermion, so that the fermion number is not conserved.

To cast the vector potential  $A_\mu(x)$  given in (3.1) to the form of the  $A_0(x) = 0$ , we make a gauge transformation  $V(x)$  on  $A_\mu(x)$  into the temporal gauge:

$$A'_0(x) = V^{-1}(x)A_0(x)V(x) + V^{-1}(x)\partial_0 V(x) = 0 \quad (3.3)$$

and define

$$\mathbf{A}'(x) = V^{-1}(x)\mathbf{A}(x)V(x) + V^{-1}(x)\nabla V(x). \quad (3.4)$$

Under this gauge transformations, the Higgs field  $\Phi$  can be transformed accordingly as

$$\Phi'(x) = V^{-1}(x)\Phi(x). \quad (3.5)$$

From equation (3.3), we can find the solution for  $V(x)$  by integration as

$$V(x) = \exp[i\tau \cdot \hat{\mathbf{x}}f(x)] \quad (3.6)$$

with

$$f(x) = \frac{\sqrt{\mathbf{x}^2}}{\sqrt{\mathbf{x}^2 + \rho^2}} \left[ \tan^{-1} \left( \frac{x_0}{\sqrt{\mathbf{x}^2 + \rho^2}} \right) + \theta(\mathbf{x}) \right], \quad (3.7)$$

where  $\theta(\mathbf{x})$  is a time-independent residual gauge freedom with respect to the  $A_0(x) = 0$  gauge. We can choose it to be constants,

$$\theta = \left(n + \frac{1}{2}\right)\pi, \quad n = 0, 1, 2, \dots \quad (3.8)$$

By substituting (3.1) and (3.2) into (3.4) and (3.5), respectively, we obtain the general spherically symmetric form for the gauge field  $\mathbf{A}'(x)$  and Higgs field  $\Phi'(x)$  as

$$\begin{aligned} \mathbf{A}'(x) &= \frac{1}{g} \left\{ a(r)(\boldsymbol{\tau} \times \hat{\mathbf{x}}) + b(r) [\boldsymbol{\tau} - (\boldsymbol{\tau} \cdot \hat{\mathbf{x}})\hat{\mathbf{x}}] + c(r)(\boldsymbol{\tau} \cdot \hat{\mathbf{x}})\hat{\mathbf{x}} \right\}, \\ \Phi'(x) &= \frac{v}{\sqrt{2}} [h(r) + i\boldsymbol{\tau} \cdot \hat{\mathbf{x}}k(r)] \Phi_0 \end{aligned} \quad (3.9)$$

with

$$\begin{aligned} a(r) &= -\frac{1}{x^2 + \rho^2} (r \cos 2f + x_0 \sin 2f) - \frac{\sin^2 f}{r}, \\ b(r) &= -\frac{1}{x^2 + \rho^2} (x_0 \cos 2f - r \sin 2f) - \frac{\sin 2f}{2r}, \\ c(r) &= -\frac{x_0}{x^2 + \rho^2} - \frac{df}{dr}, \\ h(r) &= \frac{1}{\sqrt{x^2 + \rho^2}} (-r \sin f + x_0 \cos f), \\ k(r) &= \frac{1}{\sqrt{x^2 + \rho^2}} (-r \cos f - x_0 \sin f), \end{aligned} \quad (3.10)$$

where  $x^2 = x_0^2 + r^2$ ,  $r = \sqrt{\mathbf{x}^2}$  and  $\hat{\mathbf{x}}$  is a unit three-vector in the radial direction given by  $\hat{\mathbf{x}} = \mathbf{x}/r$ .

By using (3.9), the Chern-Simons number  $N_{CS}$  in (2.10) can be given by

$$N_{CS}(x_0) = \frac{2}{\pi} \int_0^\infty r^2 dr \left[ 2c(a^2 + b^2 + \frac{1}{r}a) + (ba' - ab') \right] \quad (3.11)$$

where the prime means differentiation with respect to  $r$ . Due to  $x_0$  dependence of the functions  $a(r)$ ,  $b(r)$  and  $c(r)$ , the  $N_{CS}$  is the function of  $x_0$ . We do not now attribute any physical significance to the variable  $x_0$  and regard  $\mathbf{A}(\mathbf{x}, x_0)$  and  $\Phi(\mathbf{x}, x_0)$  as a path in the configuration space,  $x_0$  being simply a parameter along this path. In this way,  $x_0$  can describe the configuration space path as same as the  $N_{CS}$ .  $N_{CS}$  changes from 0 to 1 when  $x_0$  varies from  $-\infty$  to  $+\infty$ .



The static energy of gauge and Higgs fields for the configurations at some fixed value of the parameter  $x_0$  can be obtained by using (3.9):

$$\begin{aligned}
E(x_0) = \frac{4\pi}{g^2} \int_0^\infty dr \Big\{ & 2r^2(a^2 + b^2 + \frac{1}{r}a)^2 \\
& + r^2(a' + \frac{1}{r}a + 2bc)^2 + r^2[b' + \frac{1}{r}b - \frac{1}{r}(1 + 2ra)c]^2 \\
& + g^2v^2\{(k^2 + h^2)[1 + 2ra + r^2(2a^2 + 2b^2 + c^2)] \\
& + (1 + 2ra)(k^2 - h^2) - 4rbhk + r^2(h'^2 + k'^2) \\
& - 2r^2c(k'h - kh') + v^2\lambda r^2(h^2 + k^2 - 1)^2\} \Big\}
\end{aligned} \tag{3.12}$$

where the prime means differentiation with respect to  $r$ .  $E$  is the function of  $x_0$ . We can define parametrically the energy  $E$  as a function of  $N_{CS}$ .

By use of the functions in (3.10), one can calculate the static energy  $E$  for varying  $x_0$ . There exists a symmetric potential barrier, while  $E(x_0 = \pm\infty) = 0$  since the gauge and Higgs field tend to vacuum configuration as  $x_0 \rightarrow \pm\infty$  and  $E(x_0 = 0)$  is at the top of the barrier. The top of static energy  $E(0)$  is given by

$$E(0) = \pi^2 \left( \frac{3}{g^2} \frac{1}{\rho} + \frac{3}{8} v^2 \rho + \frac{1}{4} \lambda v^4 \rho^3 \right). \tag{3.13}$$

The top energy  $E(0)$  takes the minimum value when the instanton size  $\rho$  takes

$$\rho_0 = \frac{4}{g\sqrt{1 + \sqrt{1 + 8\alpha^2}}} \frac{1}{v} \tag{3.14}$$

where  $\alpha$  is given by  $\alpha = M_H/M_W$  and we take  $\lambda = g^2\alpha^2/8$ . Substituting (3.14) into (3.13), the minimum barrier height  $E^{\min}(0)$  is given by

$$E^{\min}(0) = \frac{\pi^2}{g} \left[ \frac{3}{\sqrt{1 + \sqrt{1 + 8\alpha^2}}} + \frac{8\alpha^2}{(1 + \sqrt{1 + 8\alpha^2})^{3/2}} \right] v. \tag{3.15}$$

We show the minimum barrier in Fig. 1 where the instanton size takes the value of (3.14) with  $\alpha = 1$ . In the following to calculate the critical mass of the fermion at which the exponential suppression for fermion number non-conservation disappears, we will use this minimum barrier for our calculation.

#### IV. THE RADIAL EQUATIONS FOR HEAVY FERMIONS

Let us now consider heavy fermions in the background fields of the electroweak instanton with the form of (3.9). To retain spherical symmetry we consider the heavy fermion doublets degenerate in mass. From the fermion Lagrangian (2.11), for each value of Euclidean time  $x_0$ , we obtain the time-independent Dirac equations for the lefthanded doublet

$$iD_0 q_L + i\sigma^i D_i q_L - f_q(\tilde{\Phi} u_R + \Phi d_R) = 0 \quad (4.1)$$

and for the righthanded singlets

$$\begin{aligned} i\partial_0 u_R - i\sigma^i \partial_i u_R - f_q \tilde{\Phi}^\dagger q_L &= 0, \\ i\partial_0 d_R - i\sigma^i \partial_i d_R - f_q \Phi^\dagger q_L &= 0, \end{aligned} \quad (4.2)$$

where  $\sigma^i$  are Pauli spin matrices. Wavefunctions  $q_L$ ,  $u_R$  and  $d_R$  depend on  $x_0$  which enters only as a parameter.

Spherically symmetric fermion fields are described by the ansatz:

$$\begin{aligned} q_L(r) &= e^{-i\epsilon t} [G_L(r) + i\sigma \cdot \mathbf{x} F_L(r)] \chi_h, \\ u_R(r) &= e^{-i\epsilon t} [G_R(r) + i\sigma \cdot \mathbf{x} F_R(r)] \chi_1, \\ d_R(r) &= e^{-i\epsilon t} [G_R(r) + i\sigma \cdot \mathbf{x} F_R(r)] \chi_2 \end{aligned} \quad (4.3)$$

with

$$\begin{aligned} \chi_h &= \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_S \begin{pmatrix} 0 \\ 1 \end{pmatrix}_I - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_S \begin{pmatrix} 1 \\ 0 \end{pmatrix}_I \right], \\ \chi_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_S, \quad \chi_2 = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_S, \end{aligned} \quad (4.4)$$

where  $S$  refers to spin,  $I$  to isospin.  $\chi_h$  is the hedgehog spinor satisfying the spin-isospin relation  $\sigma \chi_h + \tau \chi_h = 0$ .  $\chi_1, \chi_2$  are the constant spin spinors which can construct the hedgehog spinor  $\chi_h$  with isospin spinors  $\Phi_0$  and  $\tilde{\Phi}_0$ .

By using these ansatz for equations (4.1) and (4.2), we obtain the following set of four coupled first order differential equations:

$$\begin{aligned} F'_L + \left(\frac{2}{r} + 2\tilde{a}\right)F_L + (2\tilde{b} + \tilde{c})G_L - \tilde{\epsilon}G_L &= -(\tilde{h}G_R + \tilde{k}F_R), \\ G'_L - 2\tilde{a}G_L + (2\tilde{b} - \tilde{c})F_L + \tilde{\epsilon}F_L &= (\tilde{h}F_R - \tilde{k}G_R), \\ F'_R + \frac{2}{r}F_R + \tilde{\epsilon}G_R &= (\tilde{h}G_L - \tilde{k}F_L), \\ G'_R - \tilde{\epsilon}F_R &= -(\tilde{h}F_L + \tilde{k}G_L), \end{aligned} \quad (4.5)$$

where  $\tilde{r} = m_f r$ ,  $\tilde{\epsilon} = \epsilon/m_f$  and the prime means differentiation with respect to  $\tilde{r}$ . The functions  $\tilde{a}$ ,  $\tilde{b}$ ,  $\tilde{c}$ ,  $\tilde{h}$  and  $\tilde{k}$  come from the functions  $a(r)$ ,  $b(r)$ ,  $c(r)$ ,  $h(r)$  and  $k(r)$  in (3.10) respectively by variables replacement as  $\tilde{x}_0 = m_f x_0$  for  $x_0$ ,  $\tilde{r} = m_f r$  for  $r$  and  $\tilde{\rho} = m_f \rho$  for  $\rho$ . There are two parameters  $\tilde{x}_0$  and  $\tilde{\rho}$  in these equations. In the following calculation, we will limit  $\rho$  to take the values in formula (3.14) which makes the barrier height of instanton minimum. So there is only one parameter  $\tilde{x}_0$  in the equations (4.5).

To solve the eigenvalue equations (4.5) for the heavy fermions in the background field of instanton, it is required certain boundary conditions for the fermion wavefunctions. Wavefunctions  $G_L$  and  $G_R$  are finite and  $F_L$  and  $F_R$  are zero at  $\tilde{r} = 0$  and that all wavefunctions  $G_L$ ,  $G_R$ ,  $F_L$  and  $F_R$  tend to zero in the limit  $\tilde{r} \rightarrow \infty$ .

## V. ANOMALOUS DECAY OF HEAVY FERMION

Let us consider the system of a fermion doublet in the background field of electroweak instanton. As the Euclidean time  $x_0$  changes from  $-\infty$  to  $+\infty$ , the one-instanton field evolves from one vacuum to another topologically distinct vacuum. If the system starts with the filled Dirac sea plus one real fermion, then the final state has the filled Dirac sea and no real fermion, so that the fermion number is not conserved. For the fermion with small mass, the process of fermion number non-conservation is exponentially suppressed. However, if the fermion is sufficiently heavy, instead of tunneling from one vacuum to another topologically distinct vacuum, the fermion might freely move. So one can expect that rapid anomalous decay of the heavy fermion occur classically.

To investigate the possibility of unsuppressed fermion number violation in heavy fermion anomalous decay, we solve numerically the fermion eigenvalue equations (4.5) under the boundary conditions for bound state in the background field of instanton with minimum barrier. At first, we take the parameter  $\alpha = 1$  ( $M_H = M_W$ ) and the size of minimum barrier of instanton  $\rho_0$  is given by (3.14) with  $\rho_0 = 2.98/v$ . Giving a heavy fermion mass and a values for parameter  $\tilde{x}_0$ , which stand for the Euclidean time  $x_0$  for fixing fermion mass  $m_f$ , we can numerically solve the equations by computer and obtain the eigenvalue  $\tilde{\epsilon}$  and eigenfunctions  $F_L$ ,  $G_L$ ,  $F_R$  and  $G_R$ .

In the beginning, let us calculate the process of heavy fermion level crossing. There are only  $\tilde{\epsilon} = 0$  solutions in the case of  $x_0 = 0$  at which the Chern-Simons number  $N_{CS} = 1/2$ . These normalizable eigenstates with zero eigenvalue are the zero-mode of

fermion in the background fields of instanton that have discussed by authors[3,10]. To see that, we make a gauge transformation for the left-hand zero mode eigenfunctions

$$q'_L(x) = V(x)q_L(x), \quad (5.1)$$

by using the expression (3.6) for  $V(x)$  and (4.3) for  $q_L$ . We obtain

$$\begin{aligned} G'_L &= G_L \cos f + F_L \sin f, \\ F'_L &= -G_L \sin f + F_L \cos f. \end{aligned} \quad (5.2)$$

Comparing the functions  $G'_L$  and  $F'_L$  with the corresponding analytical solutions given by Rubakov[3] for the fermion zero mode, one found that it coincide with each others respectively.

We solve the fermion eigenvalue equations (4.5) for non-zero values of parameter  $x_0$  which can vary from  $-\infty$  to  $+\infty$ . Since the configurations of instanton are symmetric about  $x_0 = 0$  at which the zero mode appears, the fermion eigenvalue  $\tilde{\epsilon}$  should be antisymmetric with respect to the  $x_0 = 0$  configuration.

In Fig. 2 we represent the fermion eigenvalues  $\tilde{\epsilon}$  for dependence of the Chern-Simons number  $N_{CS}$  as the Yukawa coupling constant of heavy fermion  $f_q = 65$  in the background field of minimum barrier with  $\rho_0 = 2.98/v$  for  $M_H = M_W$ . Fig. 2 shows the whole process of fermion level crossing that is from positive-energy continuous state to negative-energy continuous state. Since  $N_{CS}$  is a function of  $x_0$ , Fig. 2 also represents the eigenvalue  $\tilde{\epsilon}$  dependence of  $x_0$  as  $N_{CS}$  varying from 0 to 1 correspond to the  $x_0$  varying from  $-\infty$  to  $+\infty$  and the gauge field configurations changing from one vacuum to another vacuum.

During the process of level crossing of heavy fermion, let us consider the total energy change of this system. In the classical approximation, the total energy is the sum of bosonic energy  $E_b$  and energy of the fermion occupying a energy level  $E_f$

$$E_t(x_0) = E_b(x_0) + E_f(x_0), \quad (5.3)$$

where  $E_b$  is given by (3.12) and  $E_f = m_f \tilde{\epsilon}$ . If the gauge and Higgs fields take their vacuum values,  $E_f$  is the mass of the fermion. During the process of fermion level crossing, we can evaluate the total energy  $E_t$  at each value of parameter  $x_0$ . The dependence of the total energy  $E_t$  on the Chern-Simons number  $N_{CS}$  along the minimum barrier of instanton with  $\rho_0 = 2.98/v$  (for  $M_H = M_W$ ) is shown in Fig. 3 for several values of Yukawa coupling constant  $f_q$  of heavy fermion. Fig. 3 describes the behaviors of total energy  $E_t$  in heavy fermion level crossing: at the beginning the total energy

tends to mass of fermion; at the top of the barrier the fermion level is at the zero mode and the total energy equal to the height of the barrier; at the end the total energy tends to the minus mass of the fermion in the Dirac sea. We found that the behaviors of total energy  $E_t$  depending on  $N_{CS}$  can be separated into three types:

At  $f_q \leq f_q^0 = 25.0$  (  $m_f^0 = 4.35$  TeV ), the state corresponding to the fermion is separated from the topologically non-trivial vacuum state by the barrier of electroweak instanton. So the fermionic state is metastable, its decay proceeds via tunneling and the decay amplitude is exponentially suppressed.

At  $f_q^0 < f_q < f_q^{\text{cr}} = 52.8$  (  $m_f^{\text{cr}} = 9.2$  TeV ), the total classical energy has a local minimum between the mass of fermion and the barrier height of instanton. Due to the barrier also exists, the anomalous decay only is by tunneling and the decay amplitude is small.

At  $f_q \geq f_q^{\text{cr}}$ , the classical total energy monotonically decreases along the path of the barrier of instanton. Hence the evolution of the system along this path is not classically forbidden. The initial state, corresponding to  $x_0$  at  $-\infty$ , describes the heavy fermion and trivial vacuum of the bosonic fields. As the parameter  $x_0 = 0$ , the bosonic fields at the top of minimum barrier with  $N_{CS} = 1/2$ , while the fermion level crosses zero and entry into the negative energy ranges. The final state, corresponding to  $x_0$  at  $+\infty$ , describes the topologically non-trivial vacuum, containing no real fermions.

So we found that, if the mass of the heavy fermion exceeds the critical value  $m_f^{\text{cr}} = 9.2$  TeV in the case of  $M_H = M_W$ , its anomalous decay might proceed without tunneling and its lifetime might be small. Within the classical approximation, we can conclude that the fermion number violation in the heavy fermion decay is not exponentially suppressed for sufficiently large  $m_f$ . This value of the critical mass of the heavy fermion is agreement with the naive estimate of ref. 3 and the variational calculation of ref. 9, but our values is exactly numerical results.

In the above calculation, we fix the mass of Higgs boson at  $M_H = M_W$  (for  $\alpha = 1$ ) with the size of instanton at  $\rho_0 = 2.98/v$  for minimum barrier height. Now we change the mass of Higgs boson for calculation. The critical fermion mass  $m_f^{\text{cr}}$  depending on the mass of Higgs boson is shown in the Fig. 4. We found that the value of the critical mass  $m_f^{\text{cr}}$  for unsuppressed fermion number violation tends to a constant for small values of Higgs mass and goes to large as the Higgs mass becoming large. In the range of weak Higgs self-coupling, the critical mass of the heavy fermion is small than 15 TeV. If the fourth generation fermion exists, due to the large mass hierarchy between generations, the heavy fourth quarks might exist the unsuppressed anomalous decay.

## VI. CONCLUSIONS

The detailed behavior of the total classical energy of heavy fermion in the background field of the electroweak instanton have been demonstrated by numerical calculation. The anomalous decay path of heavy fermion are also presented. Within the classical approximation, it is shown that the standard electroweak model with a fourth generation fermion doublet might appear the unsuppressed fermion number violation due to the heavy fermion anomalous decay. The dependence of the decay rate on the mass of the heavy fermion is extremely sharp: for masses below the critical value  $m_f^{cr}$  ( $m_f^{cr} = 9.2$  TeV for  $M_H = M_W$ ), the decay process occurs by tunneling and the decay probability is exponentially suppressed, while for masses exceeding the critical value the decay proceeds without tunneling, and the lifetime is small. The dependence of critical mass  $m_f^{cr}$  on the Higgs mass is also presented.

Unfortunately, within the classical approximation, the arguments presented in this paper are far from final conclusion. Because we have neglected the radiative corrections due to the boson loops and the contribution of the Dirac sea throughout the whole discussion. Even more to large Yukawa coupling (i.e. for large fermion mass), at present, one cannot analyze this strong coupling theories in a reliable way, so we cannot justify this approximation. Nevertheless we think that our results are the first step to study the problem of fermion number violation in the heavy fermion decay.

If the fourth generation fermion exist, it might be very heavy. The heavy fermion in the standard model will violate the vacuum stability due to the radiative corrections. So we must improve the standard model with new physics in the TeV region, for example, introducing extra bosonic fields with almost degenerate with fermions; supersymmetry, and so on. On the other hand, the fermion number violation mechanism in the standard electroweak theory is so fundamental and natural, so we hope that fermion number violation in the heavy fermion anomalous decay can appear rapidly even in the improved standard model.

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## FIGURE CAPTIONS

- FIG. 1** The minimum barrier of instanton with the size  $\rho_0 = 2.98/v$ : the energy (unit in  $v$ ) of gauge and Higgs fields is shown as a function of Euclidean time  $x_0$  (unit in  $1/v$ ). Here, we have taken  $M_H = M_W$  ( $\alpha = 1$ ).
- FIG. 2** In the minimum barrier of instanton with  $\rho_0 = 2.98/v$ , the normalized fermion eigenvalue  $\tilde{\epsilon} = \epsilon/m_f$  is shown as a function of the Chern-Simons number  $N_{CS}$  which changes from zero to one for the Yukawa coupling constant of heavy fermion  $f_q = 65$ .
- FIG. 3** The total energy (unit in  $v$ ) as a function of the Chern-Simons number  $N_{CS}$  along the minimum barrier of instanton in the case of  $M_H = M_W$  ( $\alpha = 1$ ) for Yukawa coupling constant of fermions taking  $f_q=65, 52.8, 40, 25, 10$ . We obtained  $f_q^{\text{cr}} = 52.8$  (in solid) and  $f_q^0 = 25.0$  (in dashed). The minimum barrier of instanton with  $\rho_0 = 2.98/v$  is also shown (in sparsely dotted).
- FIG. 4** The critical fermion masses for unsuppressed fermion number violation depend on the Higgs mass.



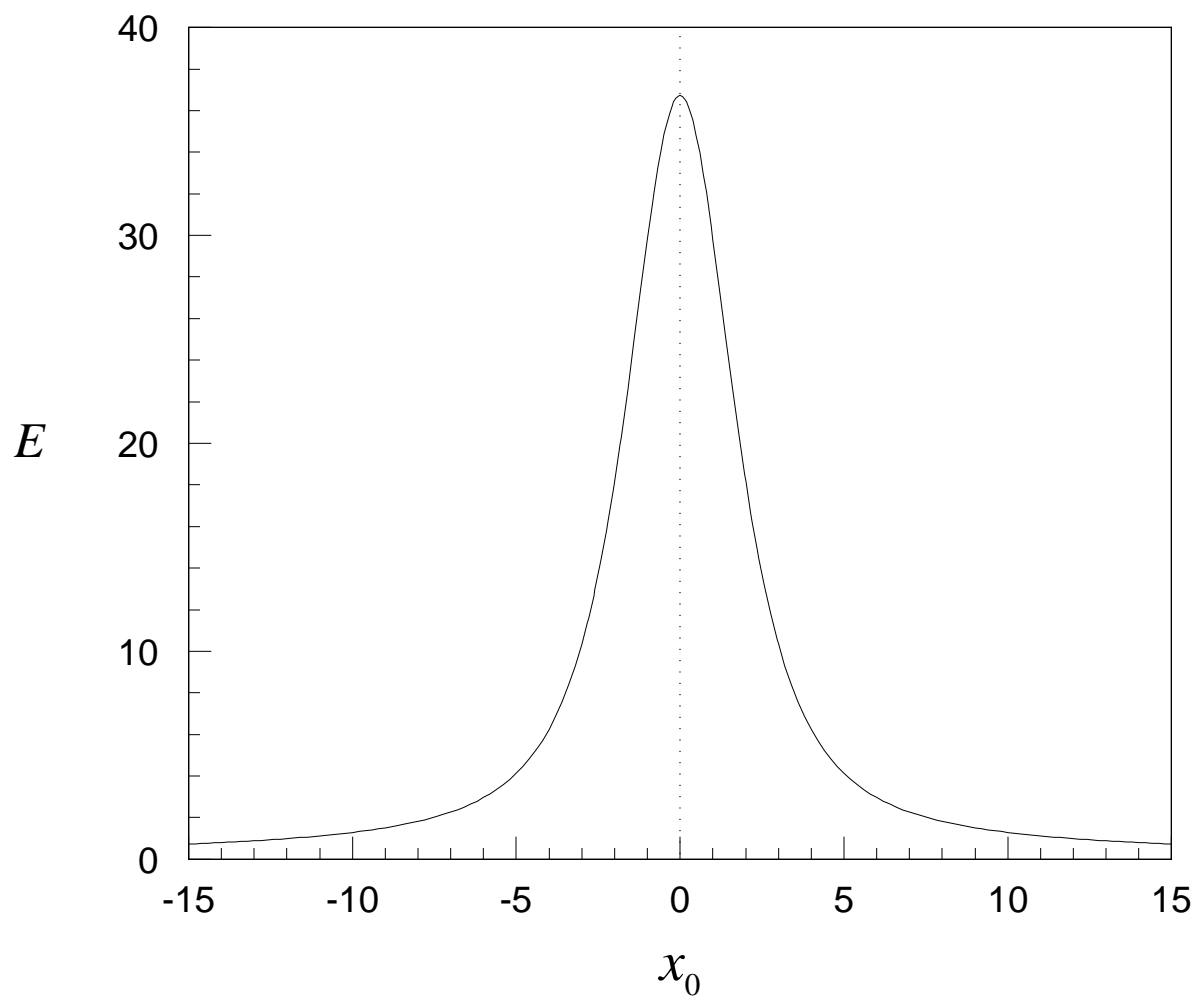


Fig. 1

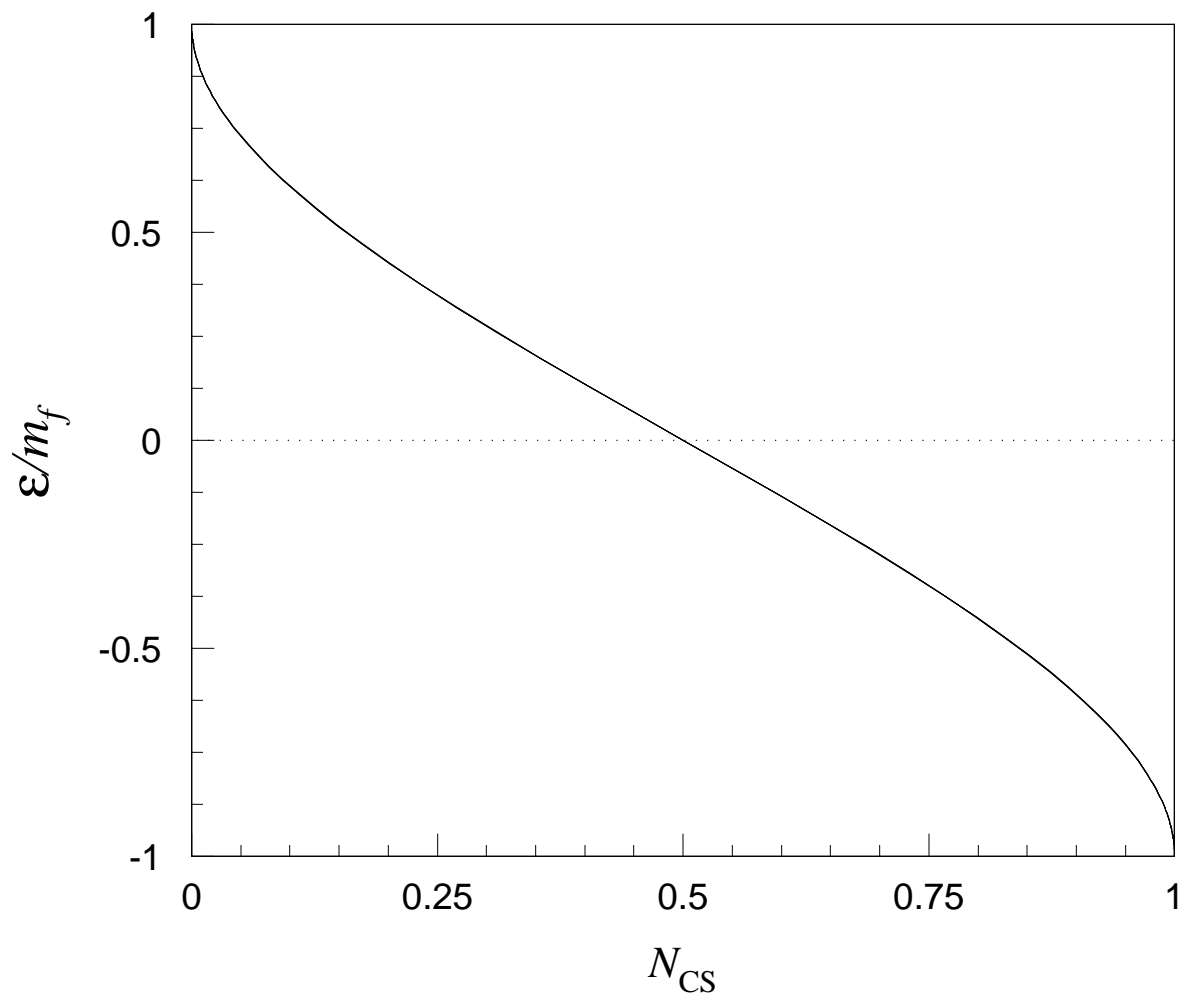


Fig. 2

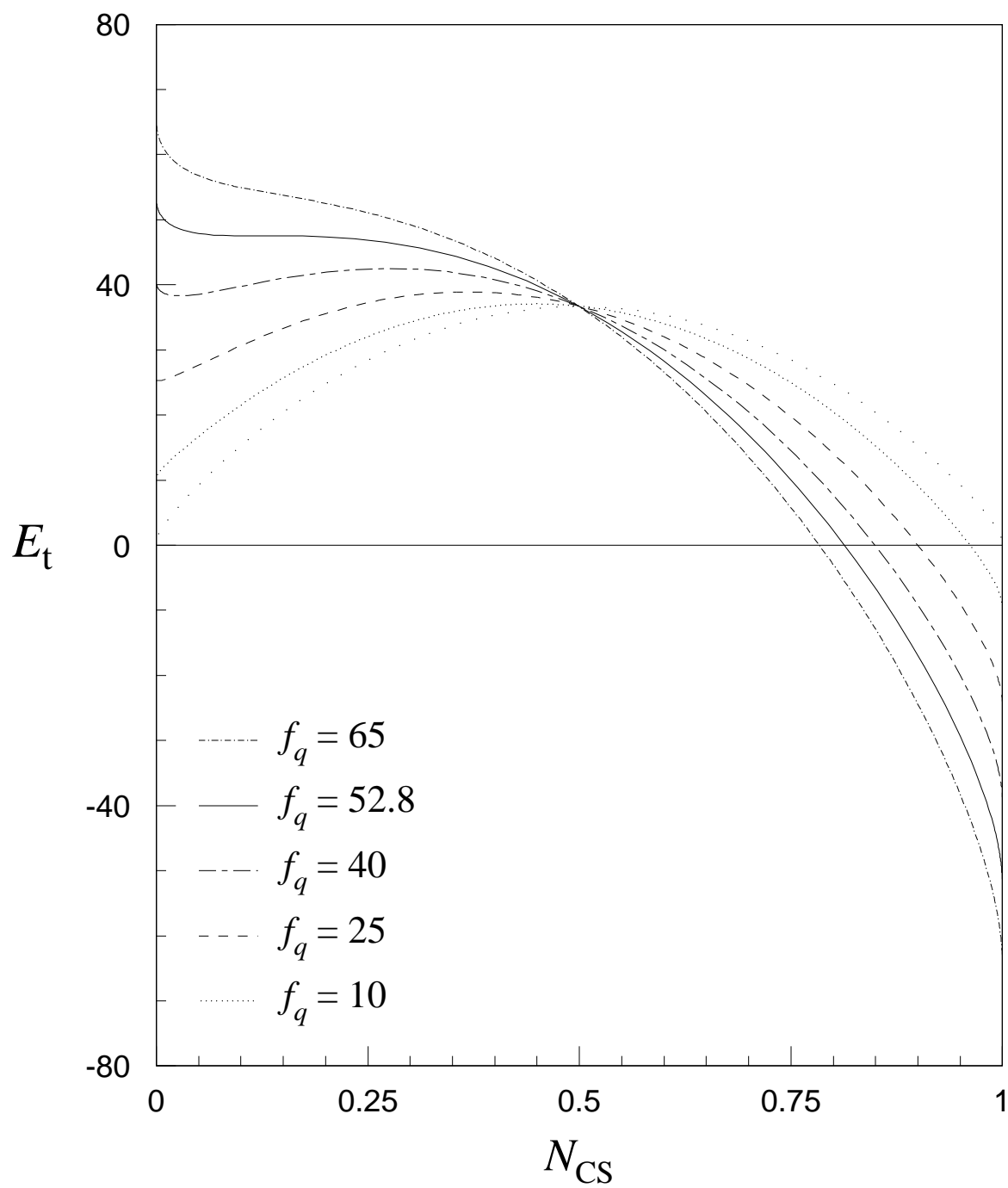


Fig. 3

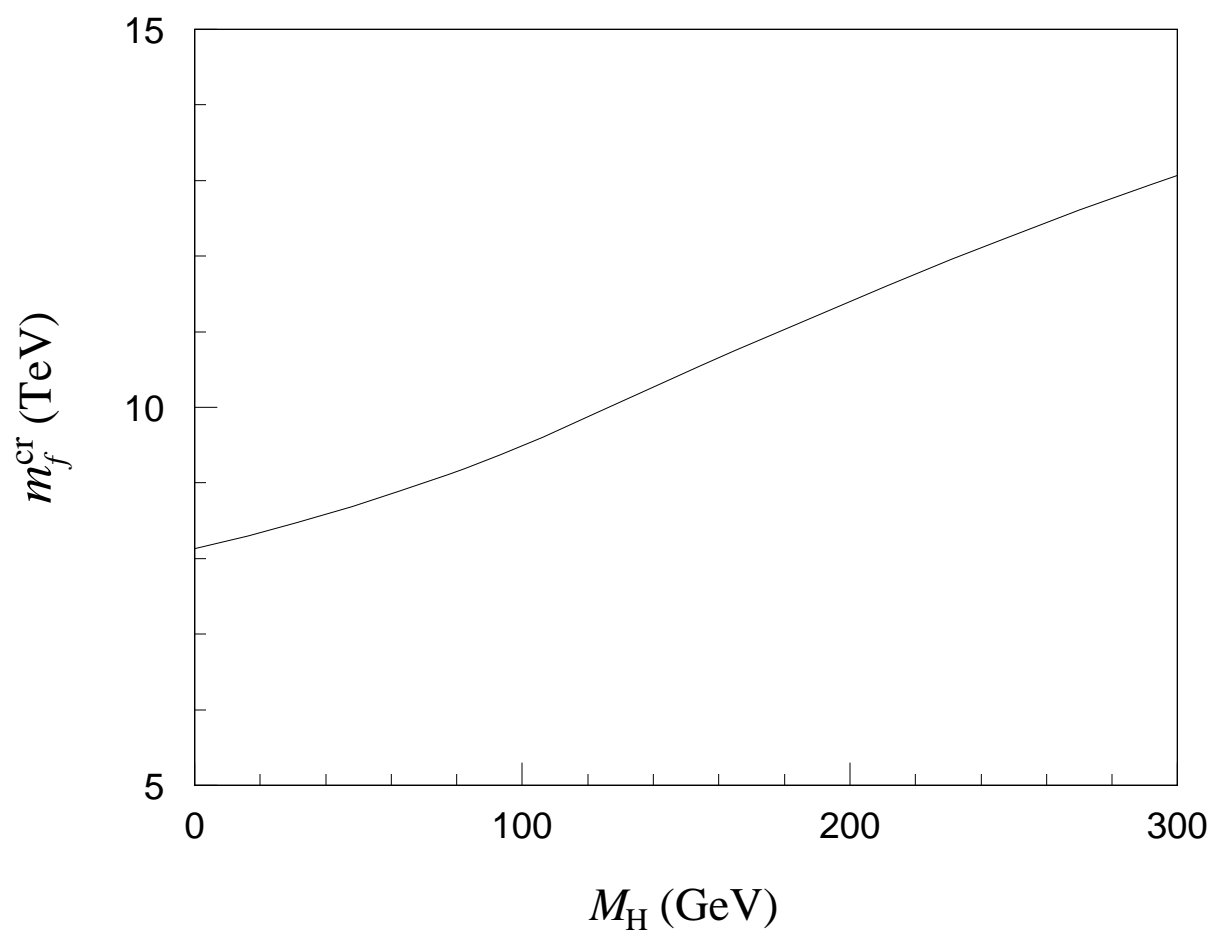


Fig. 4